

# **Design Evaluation of Safety Factors for Reflector Skeleton Made of Polymer Composites**

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## **INTRODUCTION**

Currently variety characteristics to evaluate structure remoteness from limiting state are known. These characteristics are sufficiently close by implication. However different sources use various terms to call these characteristics: «safety factor», «safety coefficient», «margin of safety», «reserve coefficient», «assurance factor» and others. In general these characteristics represent a coefficient defined by the ratio  $P^*/P$ , where  $P^*$  and  $P$  are ultimate and current values of state parameter. So, as examples of this coefficient determination one can see ratios of the yield stress to the equivalent stress of equipment elements [1], of the vector of all internal force factors in anisotropic bar section to the desired strength vector [2], of the total energy dissipation rate of the system to the external work rate [3], of the maximal shear strength to the effective shear stress on the slip surface [4], of the reactor volume (representing capacity) used for the design to the mean volume of its liquid content [5], of the elastic limit strength of spider's dragline to the spider's weight [6] and so on.

This study is dedicated to reflector antennas for terrestrial satellite communication systems working in conditions of  $Q/K_a$  frequency band. The reflector diameter is in the range 9 to 12 m and the ratio of the focal length to the diameter in the range 0,3 to 0,5.

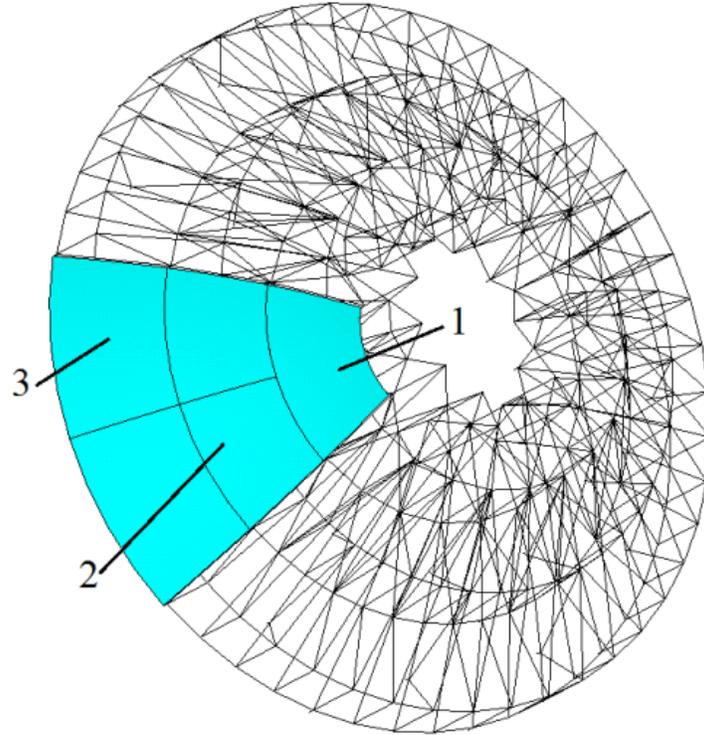
The typical reflector structural variant consists of spatial beam skeleton and mirror reflective segments of three form factors. Five segments are united in one sector (*Fig. 1*) and the mirror contains six sectors. Both skeleton and reflective segments are made of polymer composites. The master loads contain gravity force, wind and thermal impacts. The research task is to substantiate list and quantitative values of safety factors for the reflector taking into account its range of working loads and impacts.

## PROBLEM FORMULATION

The standard deviation  $\delta$  of mirror reflective surface is defined as mean value of reflective surface squared deviation  $d(r)$  from ideal parabolic surface  $S$ :

$$\delta = \sqrt{\frac{1}{S} \int d^2(r) dS},$$

where  $r = r(x, y, z)$  – radius-vectors of reflective surface points. The integral is defined over the surface of ideal parabolic surface.



**FIGURE 1.** Configuration of reflector skeleton with mounted five reflective segments of one sector: 1, 2, 3 – segments of three form factors

The wind loads are defined by the results of numerical solution of the problems of reflector external aerodynamics. Unknown components of wind velocity and pressure are determined from equations

$$\left( A + C + \frac{1}{e} HM_p^{-1} H^T \right) v^{n+1} = F + Hp^n, \quad p^{n+1} = p^n - \frac{1}{e} HM_p^{-1} H^T v^{n+1},$$

where  $A$ ,  $C$  – diffusive and convective matrices;  $H$  – pressure gradient matrix;  $M_p$  – mass matrix for the pressure;  $v^0$ ,  $p^0$  – known values of velocity and pressure

vectors;  $e > 0$  – penalty parameter;  $n \geq 0$  – iteration of the algorithm;  $F$  – bulk force vector.

Temperature field  $T$  are defined by results of numerical solution of equation of radiation and conductive heat exchange

$$K_T T_\infty + R(T) = - \int_{S_T} A_n q^n \Phi dS_T + \int_{S_T} \sigma_0 \varepsilon_0 \Phi dS_T,$$

where  $K_T$  – thermal conductivity matrix;  $T_\infty$  – ambient space temperature;  $R(T)$  – radiation vector;  $A_n$  – energy absorption coefficient;  $q^n$  – normal component of heat flow density vector;  $\Phi$  – temperature shape function vector;  $S_T$  – heat-exchange surface;  $\sigma_0$  – Stefan-Boltzmann constant;  $\varepsilon_0$  – degree of blackness of the surface.

The above equations jointly with three-dimensional equations of thermoelasticity are formulated for the discrete model of reflector represented as a finite element system. Solution of these equations is obtained taking into account boundary conditions of nave box mounted on the rotary support. Computed fields of internal force factors, stress and strains for load-bearing elements of the reflector are obtained a result of the solution.

Numerical analysis of stress and strain of reflector under specified loads and impacts indicated possibility in principle of appearance following limiting states:

the standard deviation  $\delta$  of mirror reflective surface exceeds allowable level  $[\delta]$  due to deforming reflector load-bearing structure;

pressure load  $F_s$  in skeleton beam elements exceeds the critical load value for the bar with the same geometry and stiffness parameters;

axial stress  $\sigma$  in skeleton beam elements exceeds ultimate strength  $\sigma_c$  of structural material.

These limiting states correspond to following safety factors: safety factor for standard deviation of mirror reflective surface  $n_\delta = [\delta] / \delta$ ; safety factor for critical force  $n_b = F_c / F_s$ ; safety factor for ultimate strength  $n_\sigma = \sigma_c / \sigma$ .

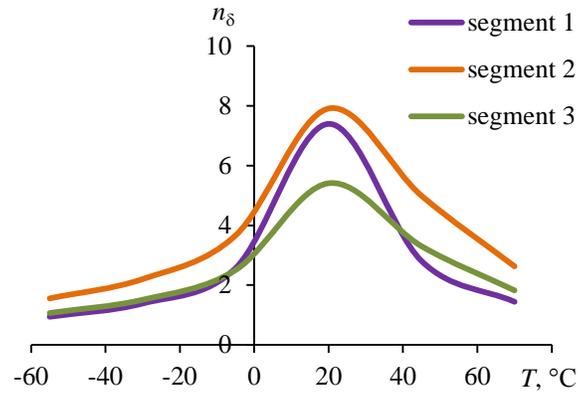
## NUMERICAL RESULTS

During evaluation safety factors on the base of numerical analysis reflector stress and strain state the following limiting values were applied. On the assumption of engineering specification for the designed reflector the allowable level of the standard deviation of the mirror reflective surface is equal to  $[\delta] = 0,25$  mm. Based on strength experimental research of the applied polymer composites, the ultimate strength is equal to  $\sigma_c = 600$  MPa. Taking into account structural design of joint junction of skeleton beam elements allowed to define critical load by means of formula

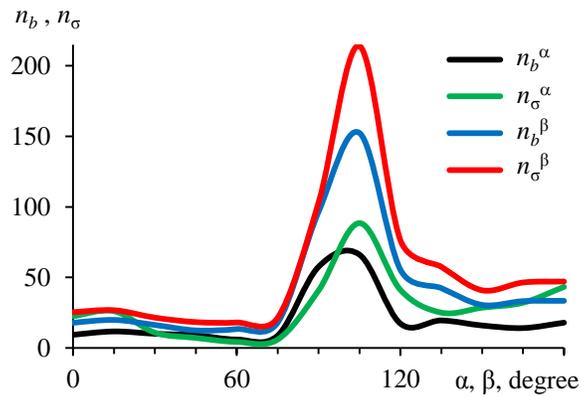
$$F_e = \frac{\pi^2 EI}{(\mu l)^2},$$

where  $E$  – Young's modulus for beam material;  $I$  – minimum inertia moment for beam cross section;  $\mu$  – coefficient of length reduction;  $l$  – beam length.

Based on the specified solar radiation intensity and convective heat exchange conditions normalized by the technical requirements, the dependences  $n_\delta = (T)$  of the design values of the safety factor for the standard deviation from the ambient air temperature are obtained for segments of three form factors (*Fig. 2a*).



(a)



(b)

**FIGURE 2.** Calculated dependencies of safety factors for standard deviation of mirror reflective surface (a), critical load and ultimate strength (b)

Variety of geometrical positions of reflector in the vertical plane is assigned with declivity angle  $\alpha$  of focal axis to horizon. Alteration of this angle causes reallocation of gravity forces by structure volume. Wind impacts assumed to operate parallel to earth surface at the angle  $\beta$  towards focal axis projection on the horizontal plane. Alteration of  $\alpha$  and  $\beta$  angles within the range of 0 to 180 degrees

allows to define all possible reflector positions in space and all possible combinations of gravity loads and wind impacts. The hazard of limiting states appearance under these conditions is characterized with calculated dependencies of safety factors for critical load  $n_b^\alpha = f(\alpha)$ ,  $n_b^\beta = f(\beta)$  and ultimate strength  $n_\sigma^\alpha = f(\alpha)$ ,  $n_\sigma^\beta = f(\beta)$  on angles  $\alpha$  and  $\beta$  (Fig. 2b).

## CONCLUSION

Obtained results lead to following conclusions regarding reflector structural design and the hazard of limiting states appearance.

The segment of form factor 1 is characterized with the greatest safety factors for standard deviation of mirror reflective surface but the segment of form factor is characterized with the least ones in the range of operational temperatures. This is because of the difference in segments sizes: smaller segment size leads to enlargement safety factors. The largest safety factors arise at a temperature of 20 Celsius degree but the least ones are associated with negative ambient space temperatures.

It tends to be the rule that the safety factor for ultimate strength is large than the safety factor for critical load. It means the hazard of buckling collapse of skeleton beam elements exceeds the hazard of elements fracture.

Alteration of the angle  $\beta$  in comparison with alteration of the angle  $\alpha$  causes larger dispersion and large values of safety factors. Response of calculated safety factors to alteration of the angle  $\alpha$  is insignificant.

The least values of safety factors for critical load and ultimate strength arise in circumstances in which angles  $\alpha$  and  $\beta$  are in the range 0 to 75 degrees, the largest ones – in the range 90 to 120 degrees.

The results obtained represent informational background for practical optimization load bearing reflector structures and serve as the basis for systems analysis of design calculations of large-size precise reflectors.

## ACKNOWLEDGMENTS

This work was done during the complex project and was financially supported by the Russian Federation Government (Ministry of Education and Science of the Russian Federation). Contract № 02.G25.31.0147.

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